MTH207 – Discrete Structures 1
Sample Questions for Exam 1
Fall 2014

1. Let A, B, and C denote three sets.
a. Give an example to show that if $A \cup B = A \cup C$ then set B need not equal set C.

b. Give an example to show that if $A \cap B = A \cap C$ then set B need not equal set C.

2. **a.** State the two De Morgan's Laws:

$$\overline{A \cap B} = \_\_\_\_\_\_\_ \text{ and } \overline{A \cup B} = \_\_\_\_\_\_\_$$

b. Use these laws to show the following:

$$\overline{\left(A \cap \overline{B}\right) \cup C} = \left(\overline{A \cup C}\right) \cup \left(B \cap \overline{C}\right).$$

3. Prove that $5^{n+1} + 2.3^n + 1$ is a multiple 8 for all positive integers n.

4. Prove the following: If x is an irrational non-negative number, then $\sqrt{x}$ is irrational.

5. If you know that the proposition $(p \wedge q) \to r \vee s$ is false, what can you say about the truth values of the propositions: $(r \vee -s) \to -p$? $p \to q \vee r \vee s$?

6. Construct the truth table of $(p \wedge q) \vee (-p \wedge q)$.

7. Show in 2 different ways the equivalence of the following statements: $p \to (q \to r)$ and $(p \wedge q) \to r$

8. Let $P(x,y)$ denote the predicate y is a multiple of x, where x and y are positive integers.
   a. Translate into English the statement $\forall x, \exists y P(x, y)$
   b. Write the negation of the statement in part a. using English.
   c. Write the negation using symbols.

9. State the converse and the contra-positive of the conditional statement: *If it is sunny tomorrow, then I will go for a walk in the woods*.

10. What is the difference between the following nested quantifications: $\forall x, \exists y P(x, y)$ and $\exists x, \forall y P(x, y)$? Give an example of a predicate $P(x, y)$ to show that $\forall x, \exists y P(x, y)$ and $\exists x, \forall y P(x, y)$ have different truth values.

11. Show that $3n+2$ is odd if and only if $9n+5$ is even.

Sample Exam – MTH 207 – Some answers

2) (b) $\overline{(A \cap \overline{B}) \cup C} = \overline{(A \cap \overline{B})} \cap \overline{C}$

$= (\overline{A} \cup \overline{\overline{B}}) \cap \overline{C} = (\overline{A} \cup B) \cap \overline{C} = (\overline{A} \cap \overline{C}) \cup (B \cap \overline{C})$

$= (\overline{A \cup C}) \cup (B \cap \overline{C})$.

3) $(5^{n+1} + 2.3^n + 1)$ is a multiple of 8, for all $n \geq 1$.

$n = 1$; $5^2 + 2.3^1 + 1 = 32$ ✓

Suppose $(5^{k+1} + 2.3^k + 1)$ is a multiple of 8

To show: $(5^{k+2} + 2.3^{k+1} + 1)$ is a multiple of 8

Indeed, $(5^{k+2} + 2.3^{k+1} + 1) = 5.5^{k+1} + 2.3^{k+1} + 1$

$= 5.(5^{k+1} + 2.3^k + 1) - 5.2.3^k - 5 + 2.3^{k+1} + 1$

$= 5.(8m) - 3^k(10 - 6) - 4$

$= 5.(8m) - 4(3^k + 1)$

Now 3^k is odd $\Rightarrow (3^k + 1)$ is even

$\Rightarrow (3^k + 1) \times 4$ is a multiple of 8.

Hence the result follows.

4) If x is irrational, then $\sqrt{x}$ is irrational

Equivalently: If $\sqrt{x}$ is rational, then x is rational.

$\sqrt{x}$ is rational $\Rightarrow \sqrt{x} = \frac{m}{n} \Rightarrow x = \frac{m^2}{n^2}$, a rational number.

) If $(p \wedge q) \rightarrow (r \vee s)$ is false, then

$(p \wedge q)$ is True & $(r \vee s)$ is false

$\therefore$ p is T, q is T, r is F, & s is F.

(you can now proceed from here)

) ✓ easy

) One way is the truth table.

2nd Method: $p \rightarrow (q \rightarrow r) \equiv \neg p \vee (q \rightarrow r)$

$$\equiv \neg p \vee (\neg q \vee r)$$

$$\equiv (\neg p \vee \neg q) \vee r$$

$$\equiv \neg (p \wedge q) \vee r$$

$$\equiv (p \wedge q) \rightarrow r.$$

3) (a) Every integer x has a multiple.

(b) $\exists x, \forall y \ \neg P(x,y)$

(b) There exists a +ve integer that has no multiples.

$(3n+2 \text{ odd}) \Longleftrightarrow (9n+5)$ is even.

Proof: Suppose $3n+2 = 2k+1 \Rightarrow 3n = 2k-1$

$$\Rightarrow 9n+5 = 9(3n)+5 = 9(2k-1)+5$$

$$= 18k-4 = 2(9k-2) \text{ even.}$$

Suppose now that $9n+5$ is even $\Rightarrow$, to show $3n+2$ is odd.

Do instead: $(3n+2)$ even $\Rightarrow (9n+5)$ odd.

$3n+2 = 2k \Rightarrow 3n = 2k-2 \Rightarrow 9n+5 = 3(2k-2)+5 = 6k-1$ odd ✓